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Risk analysis in investment appraisal based on the Monte Carlo simulation technique

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Abstract. This work has been prepared for the purpose of presenting the methodology and uses of the Monte Carlo simulation technique as applied in the evaluation of investment projects to analyze and assess risk. In the deterministic appraisal the basic decision rule for a project is simply to accept or reject the project depending on whether its net present value (NPV) is positive or negative, respectively. Similarly, when choosing among alternative (mutually exclusive) projects, the decision rule is to select the one with the highest NPV, provided that it is positive. Recognizing the fact that the key project variables (as: volume of sales, sales price, costs) are not certain, an appraisal report is usually supplemented to include sensitivity and scenario analysis tests. Both tests are static and rather arbitrary in their nature. During the simulation process, random scenarios are built up using input values for the project's key uncertain variables, which are selected from appropriate probability distributions. The results are collected and analyzed statistically so as to arrive at a probability distribution of the potential outcomes of the project and to estimate various measures of project risk.

PACS. 02.70.Lq Monte Carlo and statistical methods

1 The risk analysis process

The risk analysis process consists of the following steps [1]:

- *Developing a conceptual model of the problem under study.*
  This involves the creation of a forecasting model, which defines the mathematical relationships between numerical variables that relate to forecasting the future. For example we can consider simple cash flow model: Cash flow = (revenues - costs-deprecation) × (1-tax rate)+deprecation.
- *Building the simulation model.*
  This includes selection of key project variables and determining their probability distributions.
- *Verification and validation of the model.*
  Verification refers to the process of ensuring that the model is free from logical errors. Data validity includes ensuring that all input data and probability distributions are truly representative of the system being modeled.
- *Performing the experiments – generation of random scenarios based on assumption set.*
- *Analysis of the results.*

The output of the risk analysis process is a probability distribution of all possible project returns. The basic decision rule for a project appraisal using certainty equivalent values as inputs and discounted at a rate adjusted for risk is simply: to accept or reject the project depending on whether its NPV is positive or negative, respectively. Investment criteria for a distribution of NPVs are not always as clear as this. In deterministic case risk is usually accounted for by including a risk premium in the discount rate. Brealy and Mayers [2] have argued that the most appropriate discount rate to use in a project appraisal based on the Monte Carlo simulation is the risk free interest rate because any other discount rate would "pre-judge risk" in a project. Another school of thought maintains that the discount rate should include a premium for systematic (or market) risk but not for unsystematic (or project risk). It is not the purpose of this paper to analyze and discuss the various schools of thought on this subject. Nevertheless, authors believe that the most appropriate discount rate is the one used in the deterministic appraisal. Then the expected value of the probability distribution of NPV is a summery indicator of the project worth which is directly comparable (and should indeed be similar) to the NPV figure arrived at in the deterministic appraisal of the same project.

The most popular measures of risk, which we can calculate knowing probability distribution of project returns are:

- *The coefficient of variation.*
  It is the standard deviation of the projected returns of

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Table 1. Spreadsheet for forecasting cash flows.

	Years				
	1	2	3	4	5
Sales	50000	50000	50000	50000	50000
Variable costs	30000	30000	30000	30000	30000
Fixed costs	5000	5000	5000	5000	5000
Deprecation	3600	3600	1800	1800	1800
Pretax profit	11400	11400	13200	13200	13200
Tax at 40%	4560	4560	5280	5280	5280
Net profit	6840	6840	7920	7920	7920
Cash flows	10440	10440	9720	9720	31720
Initial investment	−42 000				
Discounting these cash flows as 10% gives us NPV = 9756.3					

NPV divided by the expected value. Assuming a positive expected value of NPV, the lower the coefficient of variation, the lower the project risk.

- *The probability of a negative outcome of NPV.* Generally, the project is quite safe when this probability is less than 0.2. The critical value of this measure of risk is subjective since every manager has a different degree of aversion to risk. For example, from the banker's point of view the project seems quite safe, when the probability of having a net present value below zero is less than 0.1. In the next section we present simple example, which illustrates the practical applicability of Monte Carlo simulation for investment risk analysis.

2 Example

Business investment concerns starting a new production division in an industrial enterprise. A five-year operation period is expected for this division. After this period a total replacement of technical equipment of the division must occur in order to undertake the production of new products or the division will be liquidated. The investment expenditures shall be borne in year 0 (*i.e.* in the year preceding the start-up of the production division). The discount rate has been fixed at $s = 10\%$. All amounts are given in established monetary units.

I. Investment expenditures:

1. Building purchase 20 000
2. Technical equipment purchase and assembly 12 000
3. Current assets 10 000.

II. Expected market value of the assets after 5 years of operation (please assume it is the residual value) is 22 000 monetary units.

The depreciation of the fixed assets has been agreed as follows:
In case of the building the rate is straight-line – 3% a year, in case of the machines and installations an accelerated depreciation is allowed – 25% in the first two years and 10% in the next years.

There are two major sources of uncertainty. First, sales is probabilistic. During the examined 5 years of operation management estimates that sales will be normally distributed with a mean of 50 000 and a standard deviation of:

Year	1	2	3	4	5
Standard deviation	5000	5500	6000	6500	7000

Second, variable costs are highly uncertain. Management estimates costs as 55%–70% of the value of sales, so we can assume, that the range of this percent variable is $\langle 55\%; 70\% \rangle$ and all outcomes between those values are equally likely. Under these assumptions this variable follows the uniform distribution. During the examined 5 years of operation the same fixed costs (without depreciation) at the level of 10% of the revenues have been assumed as well as the income tax rate at the level of 40%.

2.1 Solution

The first task of the project evaluation is to estimate future values of the variables, such as sales and costs. Usually we calculate cash flows based on *single value* of predicted variables so as a result we obtain a *single value* of NPV. This is a typical cash flow decision problem, usually analyzed with an ordinary spreadsheet model. An Excel deterministic model for this problem is given in Table 1 [3]:

As the result of generation of random scenarios based on assumption set defined earlier (1 000 runs) we get probability distribution of all possible results of NPV. This distribution is shown in Figure 1. The report, shown in Table 2, provides a summary of key descriptive statistical measures of NPV sample distribution.

The mean of NPV is 9620.6, with a 95% confidence interval around the mean value of 9240.236 to 10000.24. This interval provides a high degree of confidence that the mean NPV value is positive. We also see that the chance, that the NPV will be less than 0 is only 4.9%, so the

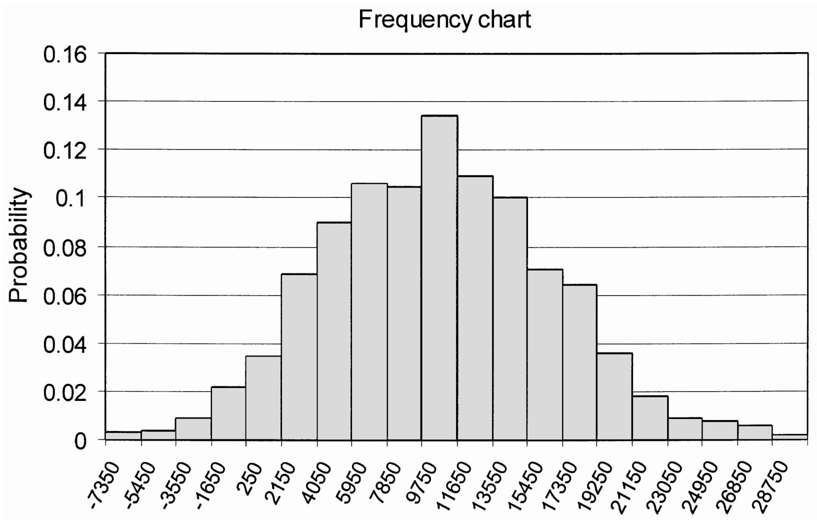

Fig. 1. Distribution of net present values.

Table 2. Summary statistics for NPV.

Mean	9620.6
Median	9603.3
Standard error of the mean	193.64
Standard deviation	5124.08
Variance	26256195.8
Skewness index	0.167957
Kurtosis	−0.0389
Coeff. of variability	53.27%
Range minimum	−8354.07
Range maximum	29217.3
Number of trials	1000
Confidence interval (95%)	380.0284

project should be excepted. In the above example for the Monte Carlo simulation Excel spreadsheet tools have been used. The flexibility of spreadsheets and their statistical capabilities make them a natural framework for simulation modeling.

3 Conclusion

Risk analysis is a useful tool enhancing the investment decision. It induces the careful re-examination of the single-value estimates in the deterministic appraisal. The difficulty in specifying range limits and probability distributions for risk analysis often resides in the fact that projected values are not adequately researched. The need to define and support explicit assumptions in the application of risk analysis therefore forces the analyst to also critically review and revise the base-case scenario.

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